Introduction

One of the most intriguing characteristics of fluvially eroded topography is its tendency to appear similar at different levels of magnification. For example, Figure 1 shows the topography in the area of Buck Creek, California, at two different magnifications. Without an external indication of scale, it is difficult to determine which representation focuses on the smaller area. This so-called "scale-invariance" has been observed in a wide range of climatic and geological conditions and over a wide range of magnifications. While this property has been exposed through empirical relations such as Hack's law and Horton's ratios, the exact nature and physical origin of topographic scale invariance have remained surprisingly obscure. In particularly, one would like know:

1. In what precise sense (or senses) is topography scale invariant?
2. What new and existing scaling laws does this property explain?
3. How does scale invariance naturally arise in river basins?
4. How can it be used for topographic simulation and interpolation?

Answering these four questions is the unifying goal of this project.

The main results are summarized in the five sections below. The first section summarizes our theoretical work on two types of scale invariance--self-similarity and multifractality--that provided a basis for addressing the questions above. The remaining four sections describe the main results for the questions above.

Self-similarity and Multifractality Theory

The need to determine a precise condition of basin scale invariance spawned theoretical work on self-similarity and multifractality. A new, simpler approach to multifractality was developed in which multifractality is treated as "stochastic self-similarity" (*i.e.* self-similarity with a random rescaling factor). Many existing multifractal results have been rederived under this framework, and several new ones have been obtained (Veneziano [8]). This approach has been the foundation of all the work on scaling issues done under this project.

Characterization of processes with given multifractal scaling has also been investigated. We have found that multifractal processes (1) have a one-to-one correspondence with certain stationary processes, (2) can be obtained as the limits of renormalized processes, and (3) can be obtained in terms of sums and products of non-scaling processes. The third characterization is important because it is most similar to the physical mechanisms that are thought to underlie scaling phenomena.

The problem of estimating multifractal scaling properties from data has been considered in Veneziano and Furcolo [9]. We have shown that a widely used method, the Double Trace Moment (DTM) method, is theoretically incorrect and produces inaccurate estimates of multifractal properties. A modified, unbiased method was developed and compared with the previous one both analytically and numerically.

Defining Topographic Self-Similarity and Multifractality

To determine an adequate definition of topographic scale invariance, we considered topography within basins and adapted existing definitions of self-similarity and multifractality. An object which is self-similar in the traditional sense is shown in part (a) of Figure 2. We call this type of self-similarity "local" because it relates features with different sizes at essentially the same location. Researchers have suggested that the elevation increments of fluvial topography are locally self-similar and have used locally self-similar models such as fractional brownian motion to simulate topography in a generic region. However, such models assume that the topography is also spatially homogenous which is not true of topography within basins. We have proposed a "global" type of self-similarity which relates large scale features near the outlet with small scale features near the sources. A simple example of global self-similarity is shown in part (b) of Figure 2, and an object with both local and global self-similarity is shown in part (c) of Figure 2.

We have developed a precise definition for the global self-similarity of basins in Veneziano and Niemann [12]. It accounts for the heterogeneity of elevation increments by rescaling about a specified origin (for example, the main stream source). The random arrangement of sub-basins is addressed by using local coordinate systems which depend on the sub-basin, and the basin boundaries are also explicitly included by constraining the relation to apply within sub-basins rather than sub-regions. Specifically, the relation can be written:

$$\{h(\underline{x}_1) - h(\underline{x}_2)\}\big|_A \overset{d}{=} r^{-H}\{h(r\underline{x}_1) - h(r\underline{x}_2)\}\big|_{r^2A}$$

where $h(\underline{x})$ is the elevation at location $\underline{x} = (x, y)$, H is a constant "self-similarity index," $r > 0$ is arbitrary, and $\overset{d}{=}$ refers to equality of the distributions. The symbols | indicate that the points on the left and right sides are required to fall within basins of sizes A and r^2A, respectively, where A is the area drained. This relation says that the elevation difference between two points $\underline{x}_1$ and $\underline{x}_2$ is statistically equal to the elevation difference between two other points $r\underline{x}_1$ and $r\underline{x}_2$ if the latter difference is multiplied by a factor r^{-H}. Figure 3 shows how this relationship applies to planar basin properties; if two sub-basins are rotated and isotropically rescaled to have the same area, then they have statistically identical shapes.

The global self-similarity condition above may not apply to all scale invariant basin topographies. A similar multifractal condition can be written if the deterministic vertical rescaling factor r^{-H} is replaced with a random variable. The cases in which each of these conditions are expected hold depend on the evolutionary dynamics and physical parameters of the basin.

New and Existing Scaling Laws

The definition of basin self-similarity is interesting in part because it explains the occurrence of many of the empirical scaling laws for river basins. In Veneziano and Niemann [13], we have

shown that the above definition of global self-similarity explains a number of these laws and predicts some new ones. For example, Hack's law relates a basin's expected main stream length L to its drained area A as: $L \propto A^{\gamma}$ where $\gamma \approx 0.57$. Because the exponent is above 1/2, Hack's law was previously thought to imply that basins become more elongated with increasing area which would imply some kind of self-affinity. However, the global self-similarity condition requires one to measure mainstream length with a ruler length that increases with basin size in order for $\gamma = 1/2$. If the mainstream length is measured with a constant ruler and the basin has both global self-similarity and some type of local fractality, $\gamma > 1/2$ is expected. Figure 4 confirms that this is the case for Buck Creek in California since γ reduces to a value very close to 1/2 when measured with a variable ruler. Similar results have been observed for the slope-area relationship. Using a variable ruler, the moments of slope vary with contributing area as expected for global self-similarity. If a constant ruler is used, the moments reveal a local multifractality.

Much previous research has examined the scaling properties of topography within generic regions rather than topography within basins. For example, the scaling properties of large scale topographic transects have been frequently studied. We have analyzed how these scaling properties are related to basin self-similarity in Veneziano and Iacobellis [10]. Because the usual analysis of topographic transects does not account for the different scaling properties between the hillslopes and the river network, spurious scaling exponents can be produced using the traditional approaches. We have shown how one can extract fluvial and hillslope scaling laws through improved analysis techniques.

Physical Origin of Self-Similarity

One would like to understand how global self-similarity (and global multifractality) arise in nature. Self-similarity can only be observed in fluvial basins if the fluvial erosion dynamics: (1) preserve self-similarity when achieved and (2) promote self-similar states from non-self-similar ones. The first requirement, which is the weaker of the two, can be addressed analytically. In Veneziano and Niemann [13], we have shown that self-similarity can be maintained in two ways--statically or dynamically. Statically self-similar topography has a form that remains frozen in time, whereas dynamic self-similar topography changes through time in a way that preserves self-similarity. Using a wide class of landscape evolution models, we derived some conditions under which either of these self-similar states may be maintained, and the self-similarity index H was calculated from the model parameters. These results were confirmed numerically with a model that operates on a radial grid in order to not distort self-similar properties. The scaling properties of models based on planar, self-affine network growth rules were also investigated (Niemann et al. [6]). Specifically, Eden and Scheidegger networks were modified to include the effects of elevation. The scaling properties of these networks became more self-similar and thus closer to those of real basins, but they still exhibited some self-affinity.

Numerical modeling was also used to investigate the attractiveness of self-similar states for various models when confronted with non-self-similar initial and boundary conditions. Niemann et al. [5] showed that basins evolving from very smooth initial surfaces may not reach self-similar

states if tectonic uplift is not active. In contrast, Moglen and Bras [2,3] showed that spatial heterogeneity in erodibility encourages a basin to reach a self-similar state.

Interpolation Application

An important application of the concepts above is for interpolation of topographic surfaces. Traditionally, topographic interpolation has been done with statistical or geometrical methods, but such approaches may produce non-physical features such as pits and overshoots. One would like to use knowledge about the basin form and its evolution to develop an improved interpolation method. For example, the river networks on the interpolated topography should have similar characteristics to those observed in nature. We have done preliminary work on using geomorphological knowledge for topographic interpolation. Our approach had two main thrusts: 1. gain improved understanding of hillslopes in physical models of basin evolution, and 2. simplify and adapt a physical model for topographic interpolation.

While the work described above focused on understanding the patterns of the fluvially eroded portion of river basins, most of the terrain belongs to the hillslopes. For this reason, one must also accurately simulate the form and extent of hillslopes. In Tucker and Bras [7], we have shown how a variety of hillslope processes affects the visual appearance of the hillslopes as well as the features of their slope-area relationship. Figure 5 shows various simulated topographies with differing active hillslope processes. Soil creep and rainsplash lead to very rounded hillslope profiles whereas threshold or pore pressure induced landsliding tend to straighten the hillslope profiles. The extent of the hillslopes, which is usually measured by the drainage density, also depends on these processes. As one increases the basin relief, for example, the increase or decrease in drainage density depends on the active processes and their parameters. In Moglen et al. [4], we have also shown how the drainage density depends on changes in climate. We have found that the sign of the resulting change in drainage density depends not only on the nature of the climate change (wetter or drier) but also on the climatic regime.

We have developed a method for topographic interpolation which incorporates a simple physical model of basin topography (Flammini et al. [1]). The model simulates topography at equilibrium under the effects of fluvial erosion and threshold activated landsliding. The procedure begins with coarsely spaced elevation data. From this data, we obtain (1) the parameters for a slope-area relationship, (2) the drainage directions between the coarse grid cells, and (3) estimates for the elevations at the fine resolution using linear interpolation. The elevations and drainage directions in flat, main valleys are accepted as final estimates for those points. For the remaining region, the elevation estimates are disregarded, and drainage directions are reassigned randomly at the fine scale using the coarse scale directions. Given the drainage directions, the slope-area relationship, and elevations of the valley floors, all other elevations can be assigned. Because these elevations may be inconsistent with the drainage directions, the drainage directions and elevations must be updated iteratively until a consistent surface is reached. In the areas where the slope-area relationship is enforced, the surface will be globally self-similar.

This model has been compared with linear interpolation. Although linear interpolation produces less error in the elevation estimates, the new interpolation procedure produces less error in

the surface gradients and roughness and it develops statistically realistic drainage patterns. Figure 6 shows an example of linearly and physically interpolated surfaces along with the observed basin. The linearly interpolated surface is clearly more smooth than the real basin, whereas the physically interpolated surface has a more realistic texture. Although additional improvements in the interpolation method are required, these results already show the benefits of interpolating topographic surfaces based on the physical processes and scaling properties of fluvial terrain.

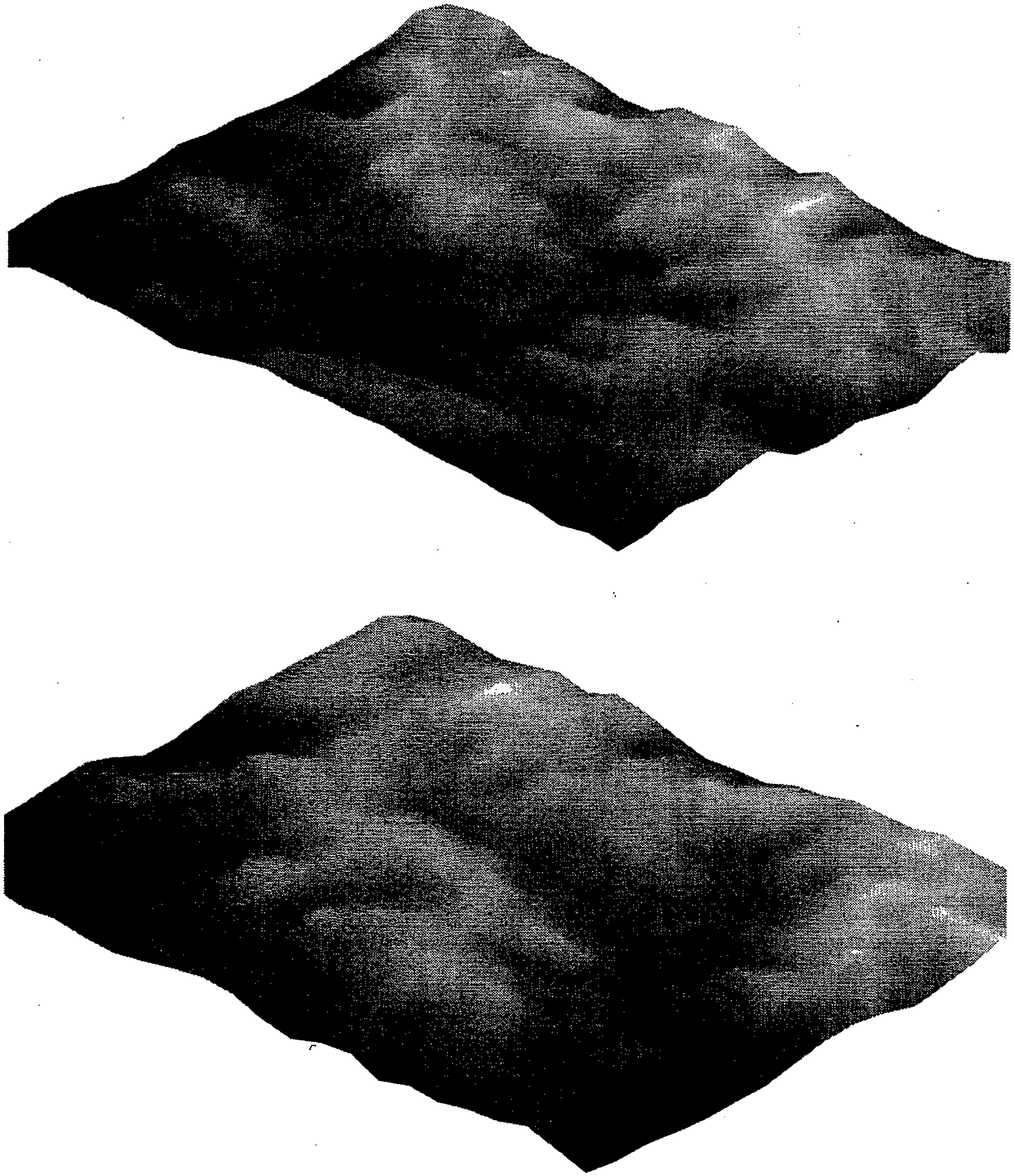

Figure 1. Two topographic surfaces from the Buck Creek area in California with shading according to elevation. The two surfaces have the same number of data points, but they have different horizontal spacings.

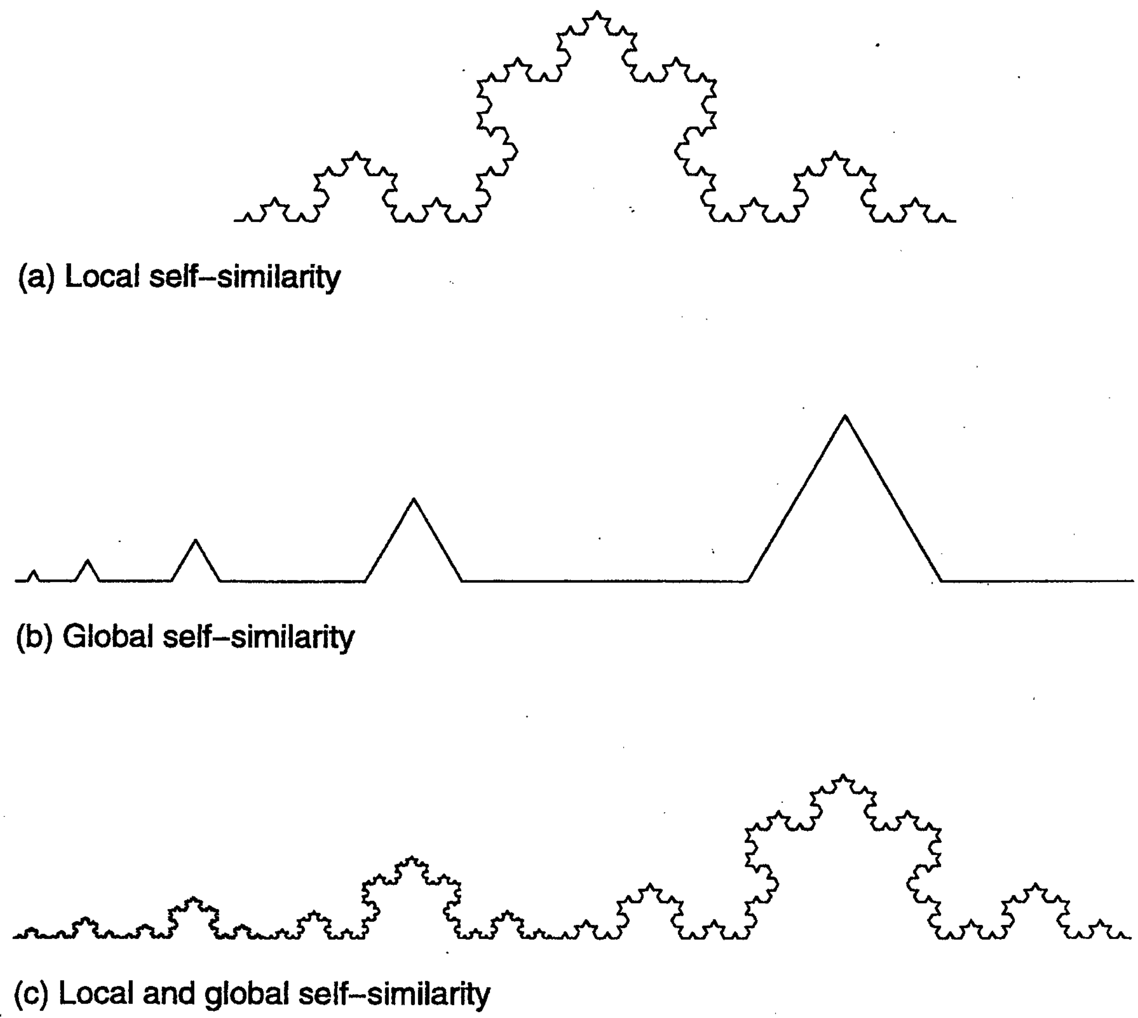

Figure 2. Examples of (a) local self-similarity (a Koch curve), (b) global self-similarity, and (c) local and global self-similarity together.

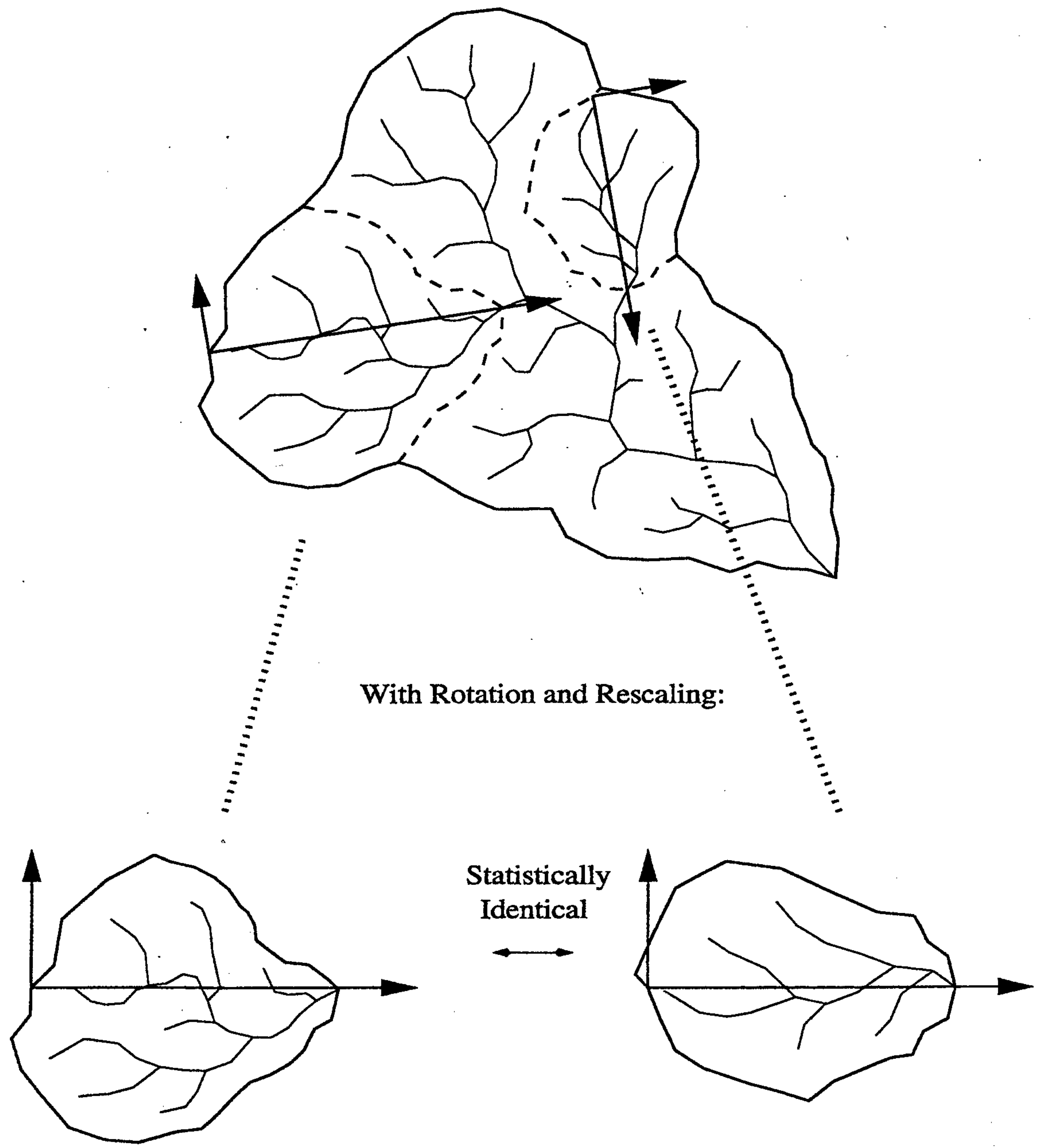

Figure 3. A schematic diagram showing the implication of the global self-similarity relation for basin shapes.

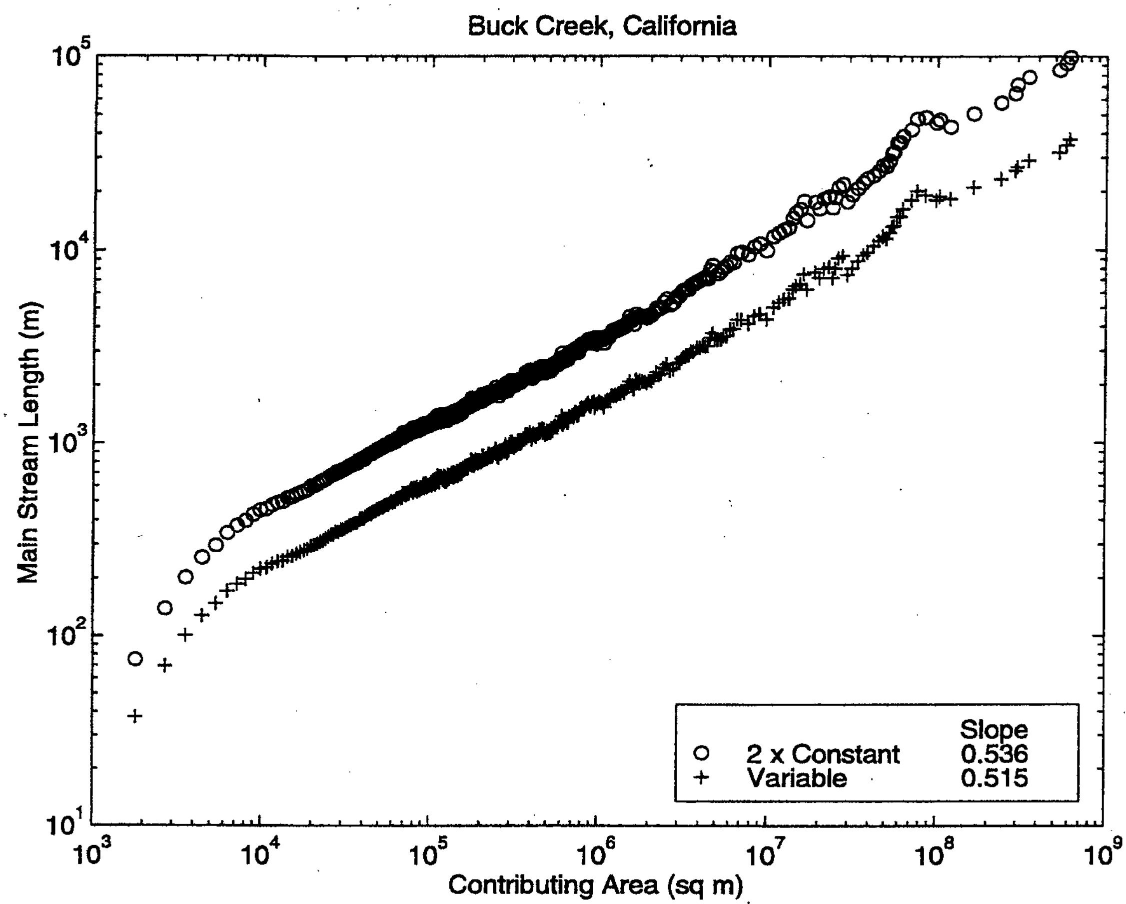

Figure 4. Hack's law measured with constant and variable rulers for Buck Creek, California. γ values are estimated from the slopes in log-log and are shown in the legend.

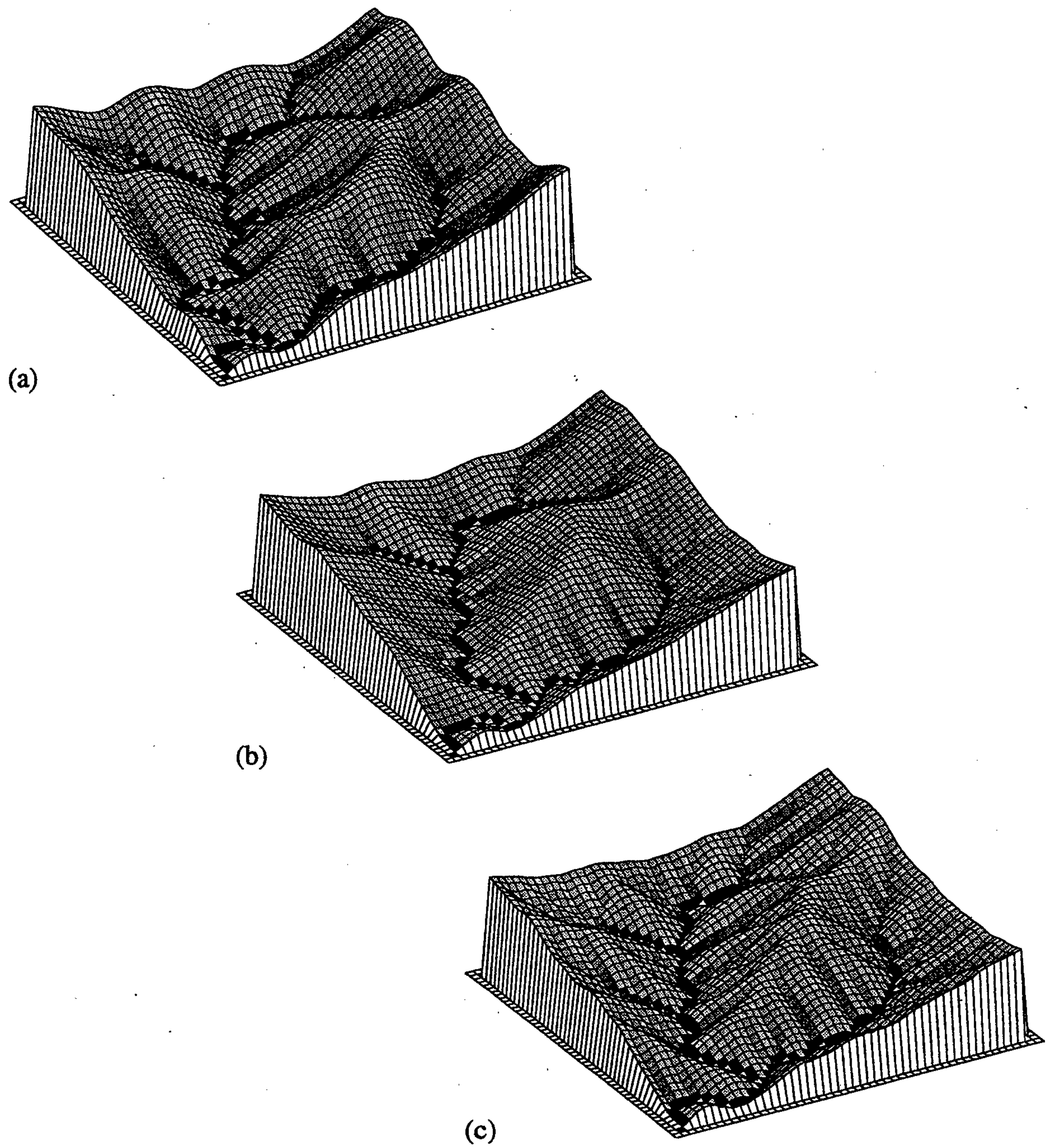

Figure 5. Simulated topographies with various hillslope processes and forms: (a) rounded hillslope profiles from soil creep (or rainsplash), (b) straightened hillslope profiles from the combination of soil creep and threshold landsliding, (c) straightened hillslope profiles with hollows from the combination of soil creep and pore pressure activated landsliding. Shading is according to contributing area.

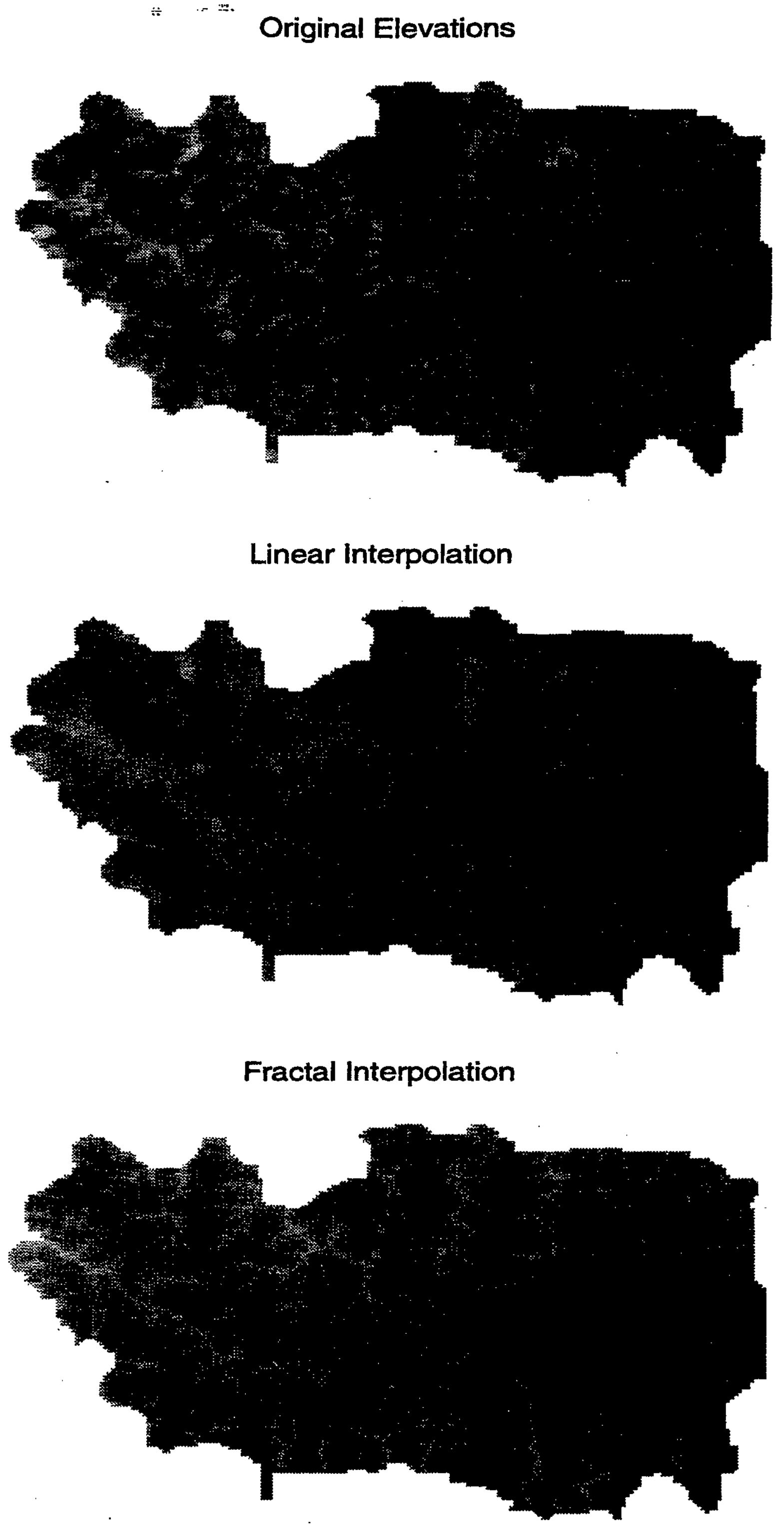

Figure 6. Interpolation algorithms applied to a sample basin: (a) the basin topography to be reconstructed, (b) the topography reconstructed using linear interpolation, (c) the topography reconstructed using fractal interpolation. Shading is by elevation.

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13. ABSTRACT *(Maximum 200 words)*

The goal of this project has been to understand the three dimensional scaling properties of terrain. In this final report, an overview of the project is given and some of the main results are summarized. The project followed five main thrusts. First, methods for the generation, characterization, and inference of multifractals were developed. Second, precise conditions for basin scaling (self-similar or multifractal) were determined. Third, it was shown that a variety of new and existing scaling laws are explained by the underlying self-similarity condition. Fourth, a wide class of detachment limited models were shown to be consistent with the basin scaling conditions under certain restrictions on the model parameters. In addition, the attraction to self-similar states was studied numerically. Fifth, a method for topographic interpolation was designed using a physical model that develops self-similar topography. These results address primary questions in geomorphology and have potential applications in areas such as terrain simulation, topographic interpolation, detection of unnatural features, and landscape restoration and management.

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